EUROPEAN UNIVERSITY INSTITUTE, FLORENCE

DEPARTMENT OF ECONOMICS

EUI WORKING PAPER No. 89/382

Spatial Price Competition With Uninformed Buyers

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** This paper was written while the second author was a Jean Monnet Fellow at the European University Institute

BADIA FIESOLANA, SAN DOMENICO (FI)

Printed in Italy in May 1989
European University Institute
Badia Fiesolana
– 50016 San Domenico (FI) –
Italy

SPATIAL PRICE COMPETITION WITH UNINFORMED BUYERS

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March 1989

ABSTRACT

In the present paper we study price rivalry among firms selling their product at different locations in space, to buyers who are imperfectly informed about prevailing prices. Spatial dispersion of sellers naturally supports the hypothesis that buyers have imperfect knowledge of prices : whence the idea of combining the spatial model of oligopolistic interaction with a search behaviour of buyers.

ISSN-0771 3894

1. INTRODUCTION

The two approaches have been initiated, the first by Hotelling (1929) and the second by Stigler (1961). Combining these two approaches, Gabszewicz and Garella (1986) have studied a model with two firms located along a linear market, with consumers who know the price quoted by their nearest seller, but ignore the price at the more distant shop. However these consumers can obtain full information about price at a cost which is positively related to the distance separating them from the supplier. The main result of that paper shows that, at a noncooperative price equilibrium, the two firms must announce the highest among the prices which induce no search from any customer.

In the following pages, we extend the above analysis to a linear market in which more than two firms are located. After having summarized the two firm-case in Section 2, we consider in Section 3 a situation with n firms symmetrically located, and characterize a noncooperative price equilibrium. Our findings confirm that the result obtained for the two firm-case extends to the n firm-symmetric case : price competition can stabilize only when all firms quote the same price, which again does not induce any customer to undertake search. In order to analyze the role played by symmetry in this result, we develop in Section 4 the equilibrium analysis with three firms asymmetrically located. In contrast, this asymmetry may generate a new type of price equilibrium, involving price dispersion and consumers' search. Having characterized the nature of a price equilibrium in a general framework, we consider in Section 5 the issue of its existence by means of an example; again we shall distinguish there between the n firm-symmetric case and the 3 firm-asymmetric case. In the last section we provide a brief summary of our findings.

2. THE CASE OF TWO FIRMS

Consider two sellers, 1 and 2, located, respectively, at points x_1 and x_2 of a linear market $[0, L]$ uniformly covered with potential customers t, $t \in [0, L]$ (see Figure 1).

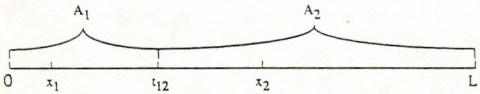

Figure 1

We shall suppose that the buyers located closer to seller 1 (the interval $[0, t_{12}] = A_1$) than to seller 2, know his price p_1, but ignore the price p_2 quoted by seller 2. Similarly, those customers located closer to seller 2 than to seller 1 (the interval $[t_{12}, L] = A_2$]) are assumed to know seller 2's, but not seller 1's price. The interval A_i is called the *natural market* of seller i, $i = 1, 2$. Finally we shall assume that, for any buyer t, it is possible to learn the unknown price by paying a search cost which increases with the distance separating this buyer from the more distant shop. On the other hand, sellers are assumed to know the information structure we have just described; furthermore, they choose their price non cooperatively.

To examine the nature of a price equilibrium (p_1^*, p_2^*), it is useful to refer to Figure 2.

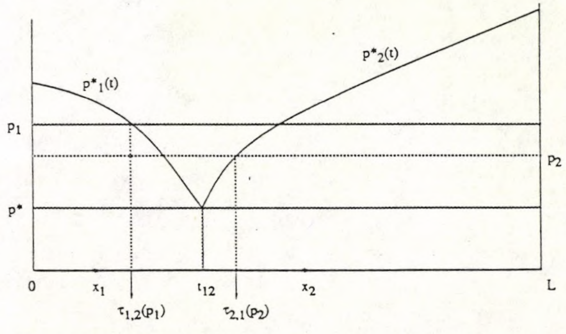

Figure 2

On this figure we have represented the highest price a consumer is willing to accept from the seller in his natural market, without searching for the other price : we call this highest price his "reservation price". Whenever the known price exceeds his reservation price, consumer t prefers to incur the information cost and postpone his decision to buy. Let $p_1^*(t)$ (resp. $p_2^*(t)$) denote the reservation price for $t \in A_1$ (resp. $t \in A_2$). We notice that the closer is a consumer t to t_{12} – the consumer at the border between the two natural markets –, the lower is his reservation price. This pattern reflects the assumption that search costs are increasing with distance : the consumers with the highest incentive to search are those located close to the border between the natural markets.

Now let us show that the only pair of prices which can constitute a price equilibrium is given by (p^*, p^*) with $p^* = p_1^*(t_{12}) = p_2^*(t_{12})$. Let us proceed by elimination. First of all, no pair of prices at which one of the sellers would quote a price strictly smaller than p^* can be an equilibrium : this seller could increase his price without loosing customers from his natural market, since none of them is searching at such a low price. This increase in price entails an increase in profits, a contradiction. Now consider a pair of prices (p_1, p_2) such as depicted on Figure 2. Denote by $\tau_{1,2}(p_1)$ (resp. $\tau_{2,1}(p_2)$) that consumer who is indifferent between buying from firm 1 (resp. firm 2) at price p_1) (resp. p_2), and searching. All those customers in the interval $]\tau_{1,2}(p_1), \tau_{2,1}(p_2)[$ decide to search and thus notice that p_2 is smaller than p_1; accordingly they buy from firm 1. At prices (p_1, p_2) demand to firm 1 (resp. firm 2) is thus equal to $D_1(p_1, p_2) = \tau_{1,2}(p_1)$ (resp. $D_2(p_1, p_2) = L - \tau_{1,2}(p_1)$). It is clear that $D_2(p_1, p_2)$ is inelastic in the domain $]p_2, p_1[$. Accordingly, seller 2 can increase his price without loosing customers, again a contradiction to the fact that such a pair of prices (p_1, p_2) would constitute a price equilibrium. This excludes any equilibrium with both prices exceeding p^* and $p_i \neq p_j$. Finally no pair of prices (p_1, p_2) with $p_1 = p_2 > p^*$ can be an equilibrium. In that case, indeed, all customers in the interval $]\tau_{1,2}(p_1), \tau_{2,1}(p_2)[$ know both prices, and can be captured by any seller who could undercut slightly the common price, thereby increasing his profit, which is not acceptable at a price equilibrium. Thus it follows from the above reasoning that *the only remaining pair of prices which can be a candidate for a price equilibrium is* (p^*, p^*). In the following section we show that this characterization of equilibrium extends to the n-firm symmetric case, i.e. when there are n firms each of which having a natural market of length $\frac{L}{n}$.

3. THE n-FIRM SYMMETRIC CASE

Consider an industry embodying n firms equally spaced at locations $x_1, \cdots, x_j, \cdots, x_n$, with $x_1 = \frac{L}{2n}$, $x_2 = x_1 + \frac{L}{n}, \cdots, x_j = x_{j-1} + \frac{L}{n}$. Symmetric locations of this kind imply that the natural market of each firm j, A_j, is of length $\frac{L}{n}$. We define also $t_{j,j+1}$ as the border between the natural markets A_j and A_{j+1}; i.e., $t_{j,j+1} = x_j + \frac{L}{2n}$.

By analogy with the previous section, we define, for $t \in A_j$, $p_j^*(t)$ as the reservation price of consumer t. We assume that $p_j^*(t)$ is a strictly increasing function of the distance $|t - x_j|$. Furthermore we assume that $p_j^*(t)$ is a continuous function of t.

From the symmetry assumption, it is easy to see that $p_j^*(t_{j,j+1}) = p_k^*(t_{k,k+1})$ for all j, k; we shall denote this common value by p^*. For convenience, denote by p^*_{-j} the $(n-1)$-dimensional vector which has p^* as components, the j^{th} component being omitted. Let us analyze the demand function $D_j(p; p^*_{-j})$ for an "interior" firm j when all its rivals quote the price p^*. Figure 3 helps understanding the following discussion.

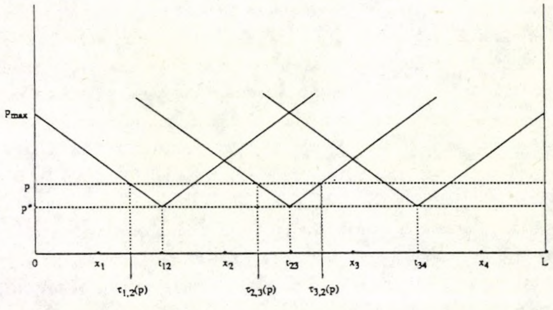

Figure 3

Clearly, for all j, $j = 2, \cdots, n-1$, $D_j(p; p^*_{-j}) = \frac{L}{n}$ if $p \leq p^*$. Then use the notation $\tau_{j,j+1}(p_j)$ (resp. $\tau_{j,j-1}(p_j)$) to represent the consumer located in seller j's natural market who, at price p_j is indifferent between buying from seller j or searching at seller $j+1$ (resp. $j-1$). Given the continuity of $p_j^*(t)$, $\tau_{j,j+1}(p_j)$ is well defined for all p_j. Let $p_{j\,\max}$ denote the smallest price at which $\tau_{j,j+1}(p) - \tau_{j,j-1}(p) = 0$,

$j = 2, \cdots, n-1$. Then, for $j = 2, \cdots, n-1$,

$$\begin{aligned} D_j(p; p^*_{-j}) &= 0, && \text{if } p \geq p_{j\,\max}; \\ &= \tau_{j,j+1}(p) - \tau_{j,j-1}(p), && \text{if } p_{j\,\max} \geq p \geq p^*; \\ &= \tfrac{L}{n}, && \text{if } p \leq p^*. \end{aligned}$$

As for the two "exterior" firms, 1 and n, the demand function – say, for firm 1, – are defined as follows for p^*_{-1} :

$$\begin{aligned} D_1(p; p^*_{-1}) &= 0, && \text{if } p \geq p_{1\,\max}; \\ &= \tau_{1,2}(p_1), && \text{if } p_{1\,\max} \geq p \geq p^*; \\ &= \tfrac{L}{n}, && \text{if } p \leq p^*. \end{aligned}$$

The demand function for firm n can be analogously defined, allowing for obvious changes in the subscripts. From continuity of $p^*_j(t)$, it follows easily that the demand functions $D_j(p; p^*_{-j})$ are also continuous, $j = 1, \cdots, n$. Further notice that, for all $p > p^*$, we have $D_1(p; p^*_{-1}) > D_j(p; p^*_{-j})$, $j = 2, \cdots, n-1$. Indeed, at each price exceeding p^*, firm 1 has no competitor on its left, while any interior firm looses customers on both sides of its natural market when it quotes a price p exceeding p^* and its neighbors quote p^*. Moreover, it follows from symmetry that all interior firms have identical demand functions $D_j(p; p^*_{-j})$, $j = 2, \cdots, n-1$.

Consider now the demand function of firm j at any $(n-1)$-price vector p_{-j}. One of the properties we shall use in the sequel is that $D_j(p; p_{-j})$ is a function of p only as long as $p > \max\{p_{j-1}; p_{j+1}\}$. In fact, when a firm quotes a price exceeding those of its neighbors, it captures only those customers from its natural market who do not search.

The profit function $\Pi_j(p; p^*_{-j})$ of an interior firm is defined by

$$\Pi_j(p; p^*_{-j}) = p \cdot D_j(p; p^*_{-j}).$$

Π_j is a continuous, bounded function on the compact interval $[0, p_{j\,\max}]$. Accordingly Π_j reaches its maximum on this interval. We shall assume henceforth that this maximum is unique and we denote it by p_{Mj}. Obviously, since $\Pi_j(p; p^*_{-j}) = p \cdot \frac{L}{n}$ is linear increasing in p in the domain $[0, p^*]$, p_{Mj} is either equal to, or strictly larger than p^*, for $j = 2, \cdots, n-1$. As for the two exterior firms, again, Π_1 and Π_n are also continuous, bounded functions on $[0, p_{1\,\max}]$ and reaches a maximum, assumed to be unique, denoted by $p_{M1} = p_{Mn}$. Let $\hat{p} = \max_{j=1,\cdots,n}\{p_{Mj}\}$. Now we state :

LEMMA 1.

i) If $\hat{p} = p^*$, then the n-tuple $(p^*, \cdots, p^*, \cdots, p^*)$ is a price equilibrium.

ii) If $(p^*, \cdots, p^*, \cdots, p^*)$ is a price equilibrium, then $\hat{p} = p^*$.

Proof. First it follows immediately from the definition of $\hat{p}$ and the assumption that $\hat{p} = p^*$ that, for any firm j, p^* is a best reply against p^*_{-j} which proves (i). Next suppose, contradicting (ii), that $(p^*, \cdots, p^*)$ is a price equilibrium and that $\hat{p} > p^*$. Then there would exist at least one firm j such that

$$\Pi_j(p_{Mj}; p^*_{-j}) > \Pi_j(p^*; p^*_{-j}),$$

a contradiction to the assumption that $(p^*, \cdots, p^*)$ is a price equilibrium. ∎

LEMMA 2. *If $\hat{p} = p^*$, then $(p^*, \cdots, p^*)$ is the unique price equilibrium.*

Proof. Suppose, contrary to the proposition, that $(\bar{p}_1, \cdots, \bar{p}_n) \neq (p^*, \cdots, p^*)$ is a price equilibrium and that $\hat{p} = p^*$. First, for at least one j, $p^* < \bar{p}_j$. Then it is readily verified that it cannot be the case that $\bar{p}_j = \bar{p}_{j+1} > p^*$ (or $\bar{p}_{j-1} = \bar{p}_j > p^*$) for any j; otherwise the two adjacent firms could undercut each other's price. It follows that at the supposed equilibrium there should be at least one firm quoting a price strictly larger than those of its neighbors. Without loss of generality, suppose it was the interior firm k; then $D_k(\bar{p}_k; \bar{p}_{-k}) = D_k(\bar{p}_k; p^*_{-k}) < D(p^*)$. But then, $\pi_k(\bar{p}_k; \bar{p}_{-k}) = p_k \cdot D(\bar{p}_k; p^*_{-k}) < p^* \cdot D(p^*)$ where the last inequality follows from the assumption that $\hat{p} = p^*$. Since playing p^* affords firm j with a profit equal to $p^* D(p^*)$, $\forall\, p_{-k}$, then one obtains a contradiction with the hypothesis that $(\bar{p}_1, \cdots, \bar{p}_n)$ is a price equilibrium. Therefore, if $\hat{p} = p^*$ no equilibrium different from $(p^*, \cdots, p^*, \cdots, p^*)$ exists, but by Lemma 1, $\hat{p} = p^*$ implies that $(p^*, \cdots, p^*, \cdots, p^*)$ is indeed an equilibrium, which completes the proof. ∎

Consider a price vector $\bar{p}$ such that $\bar{p}_1 > \bar{p}_2 \geq \bar{p}_3$. Define $\alpha(\bar{p}_1) = t_{12} - \tau_{1,2}(\bar{p}_1)$. Then, to shorten notation, for such a vector of prices and p in $]\bar{p}_2, \bar{p}_1[$, let us write $D_1(p, \bar{p}_2)$ for $D_1(p; \bar{p}_{-1})$ and $D_2(\bar{p}_1, p)$ for $D_2(p; \bar{p}_{-2})$. Figure 4 illustrates the situation corresponding to such a vector of prices.

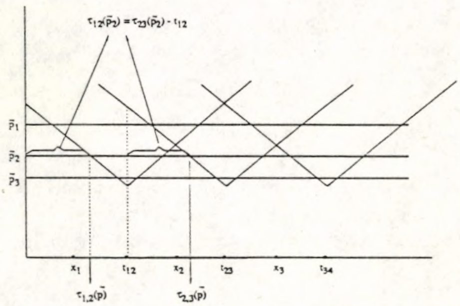

Figure 4

LEMMA 3. *For given prices $\bar{p}_1 > \bar{p}_2 \geq \bar{p}_3$ and for p in $]\bar{p}_2, \bar{p}_1[$*

$$D_1(p, \bar{p}_2) = D_2(\bar{p}_1, p) - \alpha(\bar{p}_1).$$

Proof. First notice that for $\bar{p}_2 > \bar{p}_3$ and p in $]\bar{p}_2, \bar{p}_1[$, $\tau_{1,2}(p) = \tau_{2,3}(p) - t_{12}$. Also, $\tau_{1,2} < t_{12} < \tau_{2,3}$, and

$$D_1(p, \bar{p}_2) \stackrel{\text{def}}{=} \tau_{1,2}(p).$$

Then,

$$\begin{aligned} D_2(\bar{p}_1, p) &\stackrel{\text{def}}{=} \tau_{2,3}(p) - t_{12} + t_{12} - \tau_{1,2}(p) \\ &= \tau_{1,2}(p) + \alpha(\bar{p}_1) \\ &= D_1(p, \bar{p}) + \alpha(\bar{p}_1). \end{aligned}$$

∎

The above lemma allows us to write, for $\bar{p}_1 > \bar{p}_2 \geq \bar{p}_3$ and p in $]\bar{p}_2, \bar{p}_1[$ the profit functions

$$\begin{aligned} \pi_1(p, \bar{p}_2) &= p \cdot D_1(p, \bar{p}_2) \\ \pi_2(\bar{p}_1, p) &= p \cdot D_2(\bar{p}_1, p). \end{aligned}$$

LEMMA 4. *If the n-tuple $(\bar{p}_1, \cdots, \bar{p}_n)$ is a price equilibrium, then no firm can be quoting a price strictly lower than that (those) of its neighbor(s).*

Proof. Assume that some firm j would quote a price strictly lower than its neighbors at an equilibrium. Then raising its price does not change its demand, but increases its profit, a contradiction.

LEMMA 5. *If* $(\bar{p}_1, \cdots, \bar{p}_n)$ *is a price equilibrium, then* $\bar{p}_1 = \bar{p}_2 = p^*$.

Proof. $\bar{p}_1 < \bar{p}_2$ is excluded by Lemma 4; $\bar{p}_1 = \bar{p}_2 > p^*$ is impossible because otherwise each firm would undercut each other. Then, either $\bar{p}_2 < \bar{p}_1$ or $p^* = \bar{p}_1 = \bar{p}_2$. But $\bar{p}_2 < \bar{p}_1$ is also impossible : first notice that, if $\bar{p}_2 < \bar{p}_1$, then $\bar{p}_3 < \bar{p}_2$, by Lemma 4. Furthermore, by Lemma 3,

$$\forall\ p \in]\bar{p}_2, \bar{p}_1[\ , \qquad pD_1(p_1, \bar{p}_2) = pD_2(\bar{p}_1, P_2) - p\alpha(\bar{p}_1)$$

or, equivalently,

$$\pi_1(p_1, \bar{p}_2) = \pi_2(\bar{p}_1, p_2) - p\alpha(\bar{p}_1).$$

Recall that $\pi_1(\cdot)$ and $\pi_2(\cdot)$ are continuous and single-peacked functions on $]\bar{p}_2, \bar{p}_1[$. Furthermore, by definition,

$$\bar{p}_1 = \text{argmax } \pi_1(p_1, \bar{p}_2).$$

Thus, $\forall\, \varepsilon > 0$ such that $p_2 + \varepsilon < \bar{p}_1$, one can find an $\varepsilon' > 0$ such that $\bar{p}_1 - \varepsilon' > \bar{p}_2 + \varepsilon$, satisfying

$$\pi_1(\bar{p}_1 - \varepsilon', \bar{p}_2) > \pi_2(\bar{p}_2 + \varepsilon, \bar{p}_2)$$

or, using Lemma 3,

$$\pi_2(\bar{p}_1, \bar{p}_1 - \varepsilon') - (p_1 - \varepsilon')\alpha(\bar{p}_1) > \pi_2(\bar{p}_1, \bar{p}_2 + \varepsilon) - (\bar{p}_2 + \varepsilon)\alpha(\bar{p}_1).$$

Consequently, rearranging the terms, we obtain

$$\pi_2(\bar{p}_1, \bar{p}_1 - \varepsilon') - \pi_2(\bar{p}_1, \bar{p}_2 + \varepsilon) > [p_1 - \varepsilon' - (\bar{p}_2 + \varepsilon)]\alpha(\bar{p}_1) > 0.$$

Since the inequality above is verified for each ε sufficiently small (and for each corresponding ε'), then the continuity of $\pi_2(\cdot)$ implies that

$$\pi_2(\bar{p}_1, \bar{p}_1 - \varepsilon') > \pi_2(\bar{p}_1, \bar{p}_2)$$

contradicting the hypothesis that $(\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_n)$ is a price equilibrium. ∎

LEMMA 6. *If the set of price equilibria is nonempty, it includes* $(p^*, \cdots, p^*, \cdots, p^*)$.

Proof. Let $(\bar{p}_1, \cdots, \bar{p}_n)$ be a price equilibrium. Then we know from Lemma 5 that $\bar{p}_1 = \bar{p}_2 = p^*$. Let us show that $p_2 = p^*$ is a best reply against $p_1 = p_3 = p^*$. First, for all $p > p^*$, $pD_1(p, p^*) > pD_2(p^*, p, p^*)$ since $D_1(p, p^*) > D_2(p^*, p, p^*)$. Furthermore, by symmetry, $D_1(p^*, p^*) = D_2(p^*, p^*, p^*) = \frac{L}{n}$. Since p^* is a best reply for firm 1 when firm 2 quotes p^*, a fortiori, firm 2 cannot increase its profits if firms 1 and 3 quote p^*. This argument can be extended by symmetry to all interior firms. Again by symmetry between firm 1 and n, $p_n = p^*$ is a best reply for firm n against $p_{n-1} = p^*$. ∎

We may now state the main result of this section :

PROPOSITION 1. *For the n-firm symmetric case, if there exists any price equilibria, it is unique and equal to* $(p^*, \cdots, p^*)$.

Proof. By Lemma 6, if there exists any price equilibria, $(p^*, \cdots, p^*)$ is one of them. Then by Lemma 1(ii), we know that $\hat{p} = p^*$ so that Lemma 2 applies, which concludes the proof.

4. THE THREE-FIRM ASYMMETRIC CASE

Now we leave the realm of symmetry to consider a situation with three firms asymmetrically located on the linear market $[0, L]$. As we shall see in the sequel some price equilibria can then appear involving market shares which no longer coincide with the natural markets. It is in accordance with intuition that, among the three, an exterior firm with a very large natural market could be induced, at an equilibrium, to serve only the customers located far apart from its rivals, at a high price, rather than engage in price competition with other firms in an attempt to keep its whole natural market. As for the two remaining sellers, however, Bertrand-like competition enforces their prices to descend to the level which induces no search. This leads to an equilibrium price configuration with one exterior firm quoting a rather high price and the two remaining firms announcing p^*. Of course this does not preclude another type of equilibrium price configurations in which each firm keeps its own natural market.

We shall show that those two types of equilibria, in fact, exhaust all the possibilities. The two types of price configurations thereby resulting are depicted in Figures 5 and 6.

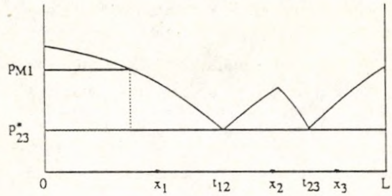

Figure 5

To proceed more rigorously, denote by p^*_{12} (resp. p^*_{23}) the reservation price of the customer located at the border between A_1 and A_2 (resp. A_2 and A_3), i.e. $p^*_{12} = p^*_1(t_{12})$ (resp. $p^*_{23} = p^*_2(t_{23})$). In general p^*_{12} and p^*_{23} need not coincide, the only exception occuring when the two exterior firms are symmetrically located around the interior firm 2, i.e. when $x_2 - x_1 = x_3 - x_2$.

The following proposition characterizes the two types of equilibrium we have just described :

PROPOSITION 2. *Let (p^*_1, p^*_2, p^*_3) be a price equilibrium, then it is of one of the two following types :*

type 1 $\quad (p^*_1, p^*_2, p^*_3) = (p^*_{12}, p^*_{12}, p_{M3})$[or $(P_{M1}, p^*_{23}, p^*_{23})$]

type 2 $\quad (p^*_1, p^*_2, p^*_3) = (p^*_{12}, p^*_{12}, p^*_{23})$ [or $(p^*_{12}, p^*_{23}, p^*_{23})$].

Proof. We proceed, again, by elimination. Notice, to start, that we cannot have $p^*_2 < p^*_3$ and $p^*_2 < p^*_1$ simultaneously, since otherwise a price increase by firm 2 increases its profits, a contradiction. Accordingly, one of the following two cases must occur : (a) $p^*_2 = p^*_1$, or (b) $p^*_2 = p^*_3$. If the first case applies we must have $p^*_1 = p^*_2 = p^*_{12}$ and in the second we must have $p^*_2 = p^*_3 = p^*_{23}$. In case (a) firm 3 must quote either $p^*_3 = p^*_{23}$, or its "monopoly price" $p^*_3 = p_{M3}$, for only one of these two prices can contribute a best reply against its opponent's choices. In case (b), firm 1 must either quote $p_1^* = p_{12}^*$ or its "monopoly price", $p_1^* = p_{M1}$, for exactly the same reason. ∎

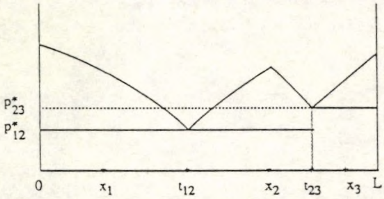

Figure 6

Notice that even the price equilibria at which no firm quotes a monopoly price, type 2 equilibria, involve price dispersion – except for the particular case in which $p_{12}^* = p_{23}^*$. The presence of price dispersion constitutes the main difference between the asymmetric and the symmetric case. The other difference is that, in equilibria of type 1, one observes that some consumers do engage in search, a phenomenon that never occurs with symmetric locations.

5. EXISTENCE OF EQUILIBRIA

So far we have limited ourselves to the problems of characterizing the nature of price equilibria, without posing the problem of its existence. Thus it is natural to provide some examples in which this question can be answered. This approach leads us first to specify the information costs as a function of distance, and the beliefs of the consumers about the unknown prices. This will allow us to derive the reservation prices of $p_j^*(t)$ of the consumers and, accordingly, the demand functions to the firms corresponding to these specifications.

For a consumer t, consider the distance $d_j(t)$ defined by

$$d_j(t) = |t - x_j|,$$

which expresses the distance between consumer t and firm j. Furthermore assume that the information cost $c_j(t)$ to be paid by a consumer $t \notin A_j$ for knowing the price p_j is defined by

$$c_j(t) = c \cdot |t - x_j|^\alpha$$

(recall that if $t \in A_j$, then t knows p_j with certainty).

Concerning the beliefs of the consumers, we shall assume that the customers' expectations about any unknown price, p_j, are represented by the uniform probability function $F(p) = p$ defined over the range of possible prices, taken to be equal to the unit interval $[0, 1]$.

Consequently, all consumers have identical uniform expectations with respect to all firms' prices.

Let $\bar{p}_j$ denote the price quoted by firm j and consider a consumer t in the natural market A_j. The expected gain $\varphi_{j,k}(\bar{p}_j, t)$ for consumer t from searching at firm k, $k = j+1$ or $k = j-1$, when knowing $\bar{p}_j$ in his own natural market, is given by

$$\varphi_{j,k}(\bar{p}_j, t) = \bar{p}_j - [1 - F(\bar{p}_j)](\bar{p}_j + c|t - x_k|^\alpha) - \int_0^{\bar{p}_j} [p + c|t - x_k|^\alpha] f(p) dp$$

where $f(p)$ denotes the uniform density corresponding to $F(p)$.

Given our assumptions, the above expression reduces to

$$\varphi_{j,k}(\bar{p}_j, t) = \frac{p_j^2}{2} - c|t - x_k|^\alpha.$$

The reservation price $p_j^*(t)$ of consumer t can then be computed from the condition that the expected gain is equal to zero, i.e. $\varphi_{j,k}(p_j^*(t), t) = 0$, or, in our example,

$$p_j^*(t) = \sqrt{2c|t - x_k|^\alpha}. \tag{5.1}$$

Let us now consider successively the problem of existence of equilibria in the frame of the above example, first for the n-firm symmetric case and then for the three-firm asymmetric case.

5.1. The n-firm symmetric case

When firms are located symmetrically on $[0, L]$, the highest price p^* at which no consumer searches is given by

$$p^* = \sqrt{2c\left(\frac{|x_k - x_{k-1}|}{2}\right)^\alpha}$$

which, from symmetry, reduces to

$$p^* = \sqrt{2c[L/(2n)]^\alpha}. \tag{5.2}$$

We know from Proposition 1 that if an equilibrium exists. all firms must quote p^* at equilibrium. On the other hand, by Lemma 1 in Section 3. if $(p^*, \cdots, p^*)$ is a price equilibrium, then p^* must be equal to $\hat{p} = \max_{j=1,\cdots,n}\{p_{Mj}\}$. So let us compute the value of $\hat{p}$ for our particular example, so as to compare it with the value of p^* as given by (5.2).

The profit function for firm 1, $\pi_1(p_1)$ on the range $]p^*, p_{\max}]$ when all other firms quote p^* is given by

$$\pi_1(p_1) = p_1\left[\frac{3L}{2n} - \left(\frac{p}{\sqrt{2c}}\right)^{2/\alpha}\right].$$

The first order condition for a maximum in $]p^*, p_{\max}]$ is given by $3L/(2n) - (p_1/2)^{2/\alpha}(1+2/\alpha) = 0$, so that

$$\text{argmax}_{p_1\in]p^*,p_{\max}]}\pi_1(p_1) = \sqrt{2c}\left[\frac{3L/2n}{1+2/\alpha}\right]^{\alpha/2}. \tag{5.3}$$

Obviously, by symmetry among firms 1 and n, $\text{argmax}\pi_1(p_1) = \text{argmax}\pi_n(p_n)$.

For an interior firm j, it is easy to check that the demand to such a firm at price p_j is equal to twice the demand to firm 1 at the same price, minus the length L/n of the natural market, i.e.

$$D_j(p_j) = 2\tau_{12}(p_j) - L/n.$$

Using the first order condition for a maximum of $\pi_j(p_j) = p_j[2\tau_{12}(p_j) - L/n]$, one obtains

$$\text{argmax}_{p_j\in]p^*,p_{\max}]}\pi_j(p_j) = \sqrt{2c}\left[\frac{2L/n}{1+2/\alpha}\right]^{\alpha/2}, \quad j = 2, \cdots, n-1. \tag{5.4}$$

By comparison of (5.3) with (5.4) it is clear that either $\hat{p} = p^*$ or $\hat{p} = \left[\frac{2L/n}{1+2/\alpha}\right]^{\alpha/2}$.

Thus, according to Proposition 1 and Lemma 4 $(p^*, \cdots, p^*)$ is a price equilibrium if, and only if, $\hat{p} = p^*$, i.e. $p^* = \sqrt{2c[(L/2n)]^\alpha} \geq \sqrt{2c}\left[\frac{2L/n}{1+2/\alpha}\right]^{\alpha/2}$, an inequality which reduces to

$$\alpha \leq 2/3.$$

Hence, the analysis of Section 3 for the symmetric case is relevant for our specification, at least as long as the cost of information is not "too" increasing with distance. In the opposite case the incentive of firms to keep their natural market by quoting p^* is beaten by the incentive to exploit the lack of information of those consumers in their natural markets which are far apart from their rivals. When such an incentive prevails the equilibrium $(p^*, \cdots, p^*)$ is destroyed and no other equilibrium exists.

5.2. The three-firm asymmetric case

We turn now to the analysis of the three-firm asymmetric case for the same example. To simplify, however, we particularize it further by assuming that the information cost parameters c and α take up the values 2 and $\frac{1}{2}$ respectively, so that

$$c_j(t) = 2|t - x_k|^{\frac{1}{2}}.$$

Furthermore, we fix the locations for firms 1 and 3 at $x_1 = 5$ and $x_3 = 10$. Then we study the set of price equilibria as a function of the length of the market, L, and of the location x_2 of the interior firm. In particular, we shall identify in the (x_2, L)-plane the loci of points to which type 1 or type 2 equilibria correspond (see Proposition 3) and those to which no equilibria correspond.

Let (p_1^*, p_2^*, p_3^*) be an equilibrium of type 1, as characterized in Proposition 3, say $(p_1^*, p_2^*, p_3^*) = (p_{M1}, p_{23}^*, p_{23}^*)$.

Applying (5.1) to the consumer t_{23} located at the border between the natural markets A_2 and A_3, we obtain

$$\begin{aligned} p_{23}^* = p_2^*(t_{23}) &= c\left(\frac{x_3 - x_2}{2}\right)^{\alpha/2} \\ &= 2\left(\frac{x_3 - x_2}{2}\right)^{1/4}, \end{aligned}$$

where the last equality follows from the parameter values we have just specified. Now we compute p_{M1}. The profit function of firm 1 is $\pi_1(p_1) = p_1 r_{12}(p_1) = p_1 \left[x_2 - \left(\frac{p}{2}\right)^4\right]$.

Using the first order condition for a maximum, we obtain

$$p_{M1} = 2\left(\frac{x_2}{5}\right)^4.$$

One can show that, to be an equilibrium, $(p_{M1}, p^*_{23}, p^*_{23}) = \left(2\left(\frac{x_2}{5}\right)^{1/4}, 2\left(\frac{x_3-x_2}{2}\right)^{1/4}, 2\left(\frac{x_3-x_2}{2}\right)^{1/4}\right)$ must verify the following set of conditions : (i) $\frac{\partial \pi_1}{\partial p_1}\Big|_{p^*_{12}} > 0$, (ii) $\frac{\partial \pi_2}{\partial p_2}\Big|_{p^*_{23}} \leq 0$, and (iii) $\frac{\partial \pi_3}{\partial p_3}\Big|_{p^*_{23}} \leq 0$.

These conditions translate in the following set of inequalities : (i) $5x_1 > 3x_2$; (ii) $17x_2 \leq 15x_3$; (iii) $2L \leq 5x_3 - 3x_2$, which delimit in the (x_2, L)-plane the region of (x_2, L) values to which corresponds an equilibrium of type 1 with firm 1 quoting p_{M1}.

Performing a similar analysis for the equilibria of type 1 in which firm 3 quotes p_{M3} would lead to another set of similar inequalities. All these inequalities taken together delimit in the (x_2, L)-plane the region of (x_2, L) values to which corresponds a price equilibrium of type 1. This region is represented on Figure 7 by the shaded area ① (recall that we have fixed $x_1 = 5$ and $x_3 = 10$).

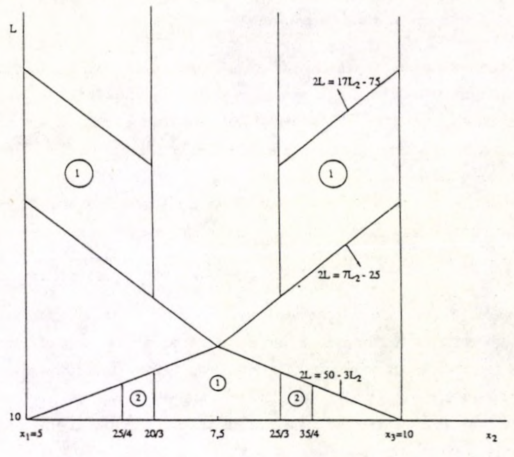

Figure 7

Applying the same methodology for identifying the set of (x_2, L) values to which a price equilibrium of type 2 corresponds, we represent on Figure 7 this set by the shaded area ②. The complement of the unions of areas ① and ② is the set of (x_2, L) pairs for which no price equilibrium exists.

As a conclusion, we notice that for the *same* information cost function the various types of equilibria can occur as a function of the asymmetries in the locations of firms.

6. CONCLUSION

In the case of a homogeneous product and if all consumers have perfect knowledge of prices, Bertrand competition drives down the price to the level of marginal cost. However there are many reasons why consumers do not enjoy from perfect information. To the extent that they are often located far away from the vendors, it is not easy for them to have at each instant a clear view of the market. Probably they catch some fragmentary aspects of it, based on past experience, or other information channels. On the other hand information diffusion is the more difficult, the further is the emission's souce of this information : it seems easy to know the prices prevailing in shops close to our living place, but it is more difficult to identify prices in shops located far away from our usual walking paths.

In the present paper we have tried to characterize price equilibria resulting from interfirm competition when buyers have imperfect knowledge of prices used by the merchants who are apart from them. We have shown that, in the case of two firms or where n firms are symmetrically located along a linear market, a price equilibrium is fully characterized by the highest among the prices which induces no search from any consumer : at that price, each seller keep all the customers in his natural market. The analysis of the three firms asymmetric case reveals however that this result relies heavily on the symmetry assumption. If the sizes of the natural markets are sufficiently different, a new type of equilibrium may appear, involving price dispersion and search. In that case an exterior firm may prefer to abandon to his neighbor some customers close to the border of his natural market, in view of capturing a higher surplus from those customers who are far apart from him. The above characterization of equilibria was obtained under the assumption that price equilibria did exist. In Section 5, we have checked on a particular example that there are, indeed, market situations in which this situation occurs.

The present model could probably be extended by making explicit the idea that the diffusion of information is more difficult, the further is the economic agent from the source where this information is emitted. Hopefully a similar characterization of price equilibria would emerge. Another possible extension would take into account the possibility for the firms of advertising the prices they quote, firms bearing the information cost rather than the customers. These extensions are beyond the scope of the present paper, but could constitute a fruitful territory for further research.

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